Starrkörperdynamik Lösungsblatt 5.2

Aufgabe 1:

Aus den Bewegungsgleichungen

$$8\ddot{\phi}_1 + 3\ddot{\phi}_2\cos(\phi_1-\phi_2) = -3\dot{\phi}_2^2\sin(\phi_1-\phi_2) - 9\omega_0^2\sin(\phi_1) \quad (1)$$

$$3\ddot{\phi}_1\cos(\phi_1-\phi_2) + 2\ddot{\phi}_2 = 3\dot{\phi}_1^2\sin(\phi_1-\phi_2) - 3\omega_0^2\sin(\phi_2) \quad (2)$$

müssen zunächst zwei Gleichungen gewonnen werden, die in den zweiten Ableitungen entkoppelt sind.

$2\cdot(1) - 3\cos(\phi_1-\phi_2)\cdot(2)$:

$$\left(16-9\cos^2(\phi_1-\phi_2)\right)\ddot{\phi}_1 = -6\dot{\phi}_2^2\sin(\phi_1-\phi_2) - 18\omega_0^2\sin(\phi_1) \\ -9\dot{\phi}_1^2\sin(\phi_1-\phi_2)\cos(\phi_1-\phi_2) \\ +9\omega_0^2\sin(\phi_2)\cos(\phi_1-\phi_2)$$

$3\cos(\phi_1-\phi_2)\cdot(1) - 8\cdot(2)$:

$$\left(9\cos^2(\phi_1-\phi_2)-16\right)\ddot{\phi}_2 = -9\dot{\phi}_2^2\sin(\phi_1-\phi_2)\cos(\phi_1-\phi_2) \\ -27\omega_0^2\sin(\phi_1)\cos(\phi_1-\phi_2) \\ -24\dot{\phi}_1^2\sin(\phi_1-\phi_2) + 24\omega_0^2\sin(\phi_2)$$

Die Transformation $z_1=\phi_1,\ z_2=\phi_2,\ z_3=\dot{\phi}_1,\ z_4=\dot{\phi}_2$ führt auf das Differenzialgleichungssystem 1. Ordnung:

$$\dot{z}_1 = z_3$$

$$\dot{z}_2 = z_4$$

$$\dot{z}_3 = \frac{-\sin(z_1-z_2)\left(9z_3^2\cos(z_1-z_2)+6z_4^2\right)+9\omega_0^2\left(\sin(z_2)\cos(z_1-z_2)-2\sin(z_1)\right)}{16-9\cos^2(z_1-z_2)}$$

$$\dot{z}_4 = \frac{\sin(z_1-z_2)\left(24z_3^2+9z_4^2\cos(z_1-z_2)\right)+3\omega_0^2\left(9\sin(z_1)\cos(z_1-z_2)-8\sin(z_2)\right)}{16-9\cos^2(z_1-z_2)}$$

Die Bahnen der Punkte *A* und *B* berechnen sich zu

$$\frac{x_A}{a} = \sin(\phi_1),\quad \frac{z_A}{a} = \cos(\phi_1)$$

und

$$\frac{x_B}{a} = \frac{x_A}{a} + \sin(\phi_2),\quad \frac{z_B}{a} = \frac{z_A}{a} + \cos(\phi_2) \quad .$$

Das Differenzialgleichungssystem wird in Octave mit der Funktion `lsode` gelöst.

<u>Programmcode für Octave:</u>

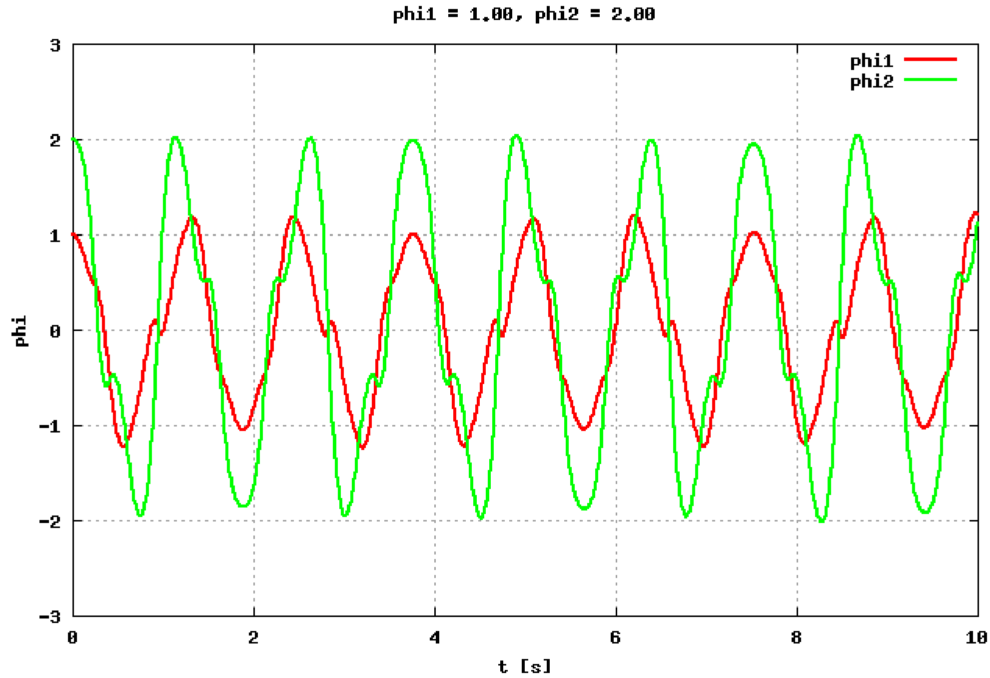
```
# Uebungsblatt 5.2, Aufgabe 1: Doppelpendel
#
# ---------------------------------------------------------

# Pendeldaten

  global omega = 40;

# Zeitraum

  t0 =    0; % Anfang
  tE =   10; % Ende
  N  = 1000; % Anzahl der Zeitschritte

  t = linspace(t0, tE, N + 1);

# Bewegungsgleichung

  function dotx = f(x, t)
     global omega;
     cs   = cos(x(1) - x(2));
     h    = 1 / (16 - 9 * cs^2);
     sn   = sin(x(1) - x(2));
     s1   = sin(x(1));
     s2   = sin(x(2));
     dotx(1) = x(3);
     dotx(2) = x(4);
     dotx3   = - sn * (9 * x(3)^2 * cs + 6 * x(4)^2);
     dotx(3) = h * (dotx3 + 9 * omega * (s2 * cs - 2 * s1));
     dotx4   = sn * (24 * x(3)^2 + 9 * x(4)^2 * cs);
     dotx(4) = h * (dotx4 + omega * (27 * s1 * cs - 24 * s2));
  endfunction

  fcn = "f";

# Anfangsbedingungen 1:
# --------------------

  x0 = [ 1.0, 2.0, 0, 0];

# Zeitintegration

  x = zeros(N + 1, 4);
  x = lsode(fcn, x0, t);
  x(:, 1:2) = rem(x(:, 1:2), 2 * pi);

  set(0, "defaultlinelinewidth", 2);
```

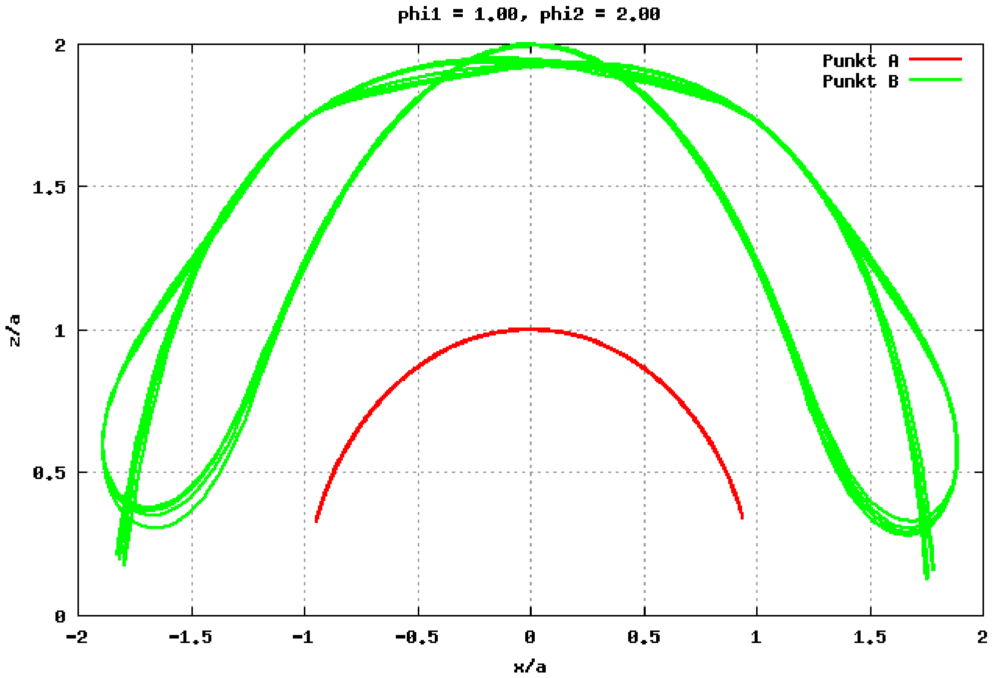
```
# Ausgabe der Winkel

  figure(1, "position", [100, 1100, 1000, 700], ...
            "paperposition", [0, 0, 5, 3]);

  plot(t, x(:, 1), "color", "red", ...
       t, x(:, 2), "color", "green");
  legend("phi1", "phi2");
  title(sprintf("phi1 = %4.2f, phi2 = %4.2f", x0(1), x0(2)));
  grid("on");
  xlabel("t [s]");
  ylabel("phi");

  print("u5_2_1_1a.png", "-dpng");

# Berechnung und Ausgabe der Bahnen

  xA =  sin(x(:, 1));
  zA =  cos(x(:, 1));
  xB = xA +  sin(x(:, 2));
  zB = zA +  cos(x(:, 2));

  figure(2, "position", [100, 100, 1000, 700], ...
            "paperposition", [0, 0, 5, 3]);

  plot(xA, zA, "color", "red", ...
       xB, zB, "color", "green");
  legend("Punkt A", "Punkt B");
  title(sprintf("phi1 = %4.2f, phi2 = %4.2f", x0(1), x0(2)));
  grid("on");
  xlabel("x/a");
  ylabel("z/a");

  print("u5_2_1_1b.png", "-dpng");

# Anfangsbedingungen 2:
# ---------------------

  x0 = [ 2.0, 3.0, 0, 0];

# Zeitintegration

  x = zeros(N + 1, 4);
  x = lsode(fcn, x0, t);
  x(:, 1:2) = rem(x(:, 1:2), 2 * pi);

# Ausgabe der Winkel

  figure(3, "position", [1100, 1100, 1000, 700], ...
            "paperposition", [0, 0, 5, 3]);

  plot(t, x(:, 1), "color", "red", ...
       t, x(:, 2), "color", "green");
```

```
  legend("phi1", "phi2");
  title(sprintf("phi1 = %4.2f, phi2 = %4.2f", x0(1), x0(2)));
  grid("on");
  xlabel("t [s]");
  ylabel("phi");

  print("u5_2_1_2a.png", "-dpng");

# Berechnung und Ausgabe der Bahnen

  xA = sin(x(:, 1));
  zA = cos(x(:, 1));
  xB = xA + sin(x(:, 2));
  zB = zA + cos(x(:, 2));

  figure(4, "position", [1100, 100, 1000, 700], ...
            "paperposition", [0, 0, 5, 3]);

  plot(xA, zA, "color", "red", ...
       xB, zB, "color", "green");
  legend("Punkt A", "Punkt B");
  title(sprintf("phi1 = %4.2f, phi2 = %4.2f", x0(1), x0(2)));
  grid("on");
  xlabel("x/a");
  ylabel("z/a");

  print("u5_2_1_2b.png", "-dpng");

# Gesamtenergie zur Kontrolle

  E = 4 * x(:, 3).^2 + x(:, 4).^2;
  E = (E + 3 * x(:, 3) .* x(:, 4) .* cos(x(:, 1) - x(:, 2)));
  E = E / 6 - 0.5 * omega * (3 * cos(x(:, 1)) + cos(x(:, 2)));
  E = E / E(1) - 1;

  figure(5, "position", [500, 500, 1000, 700]);

  plot(t, E);
  grid("on");
  xlabel("t [s]");
  ylabel("dE/E0");
```

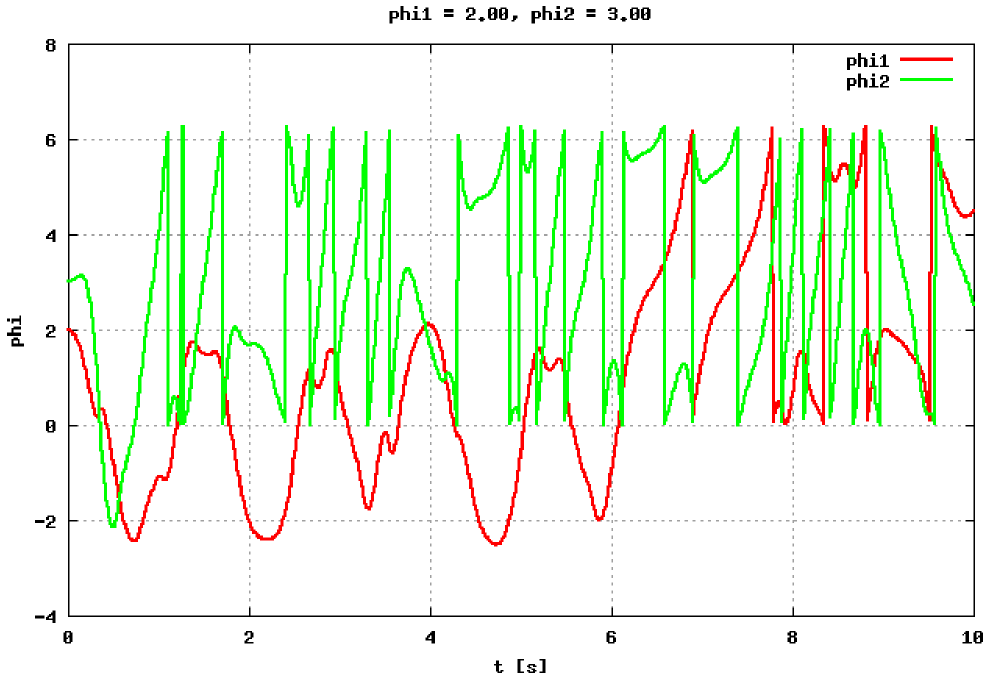
Ergebnisse:

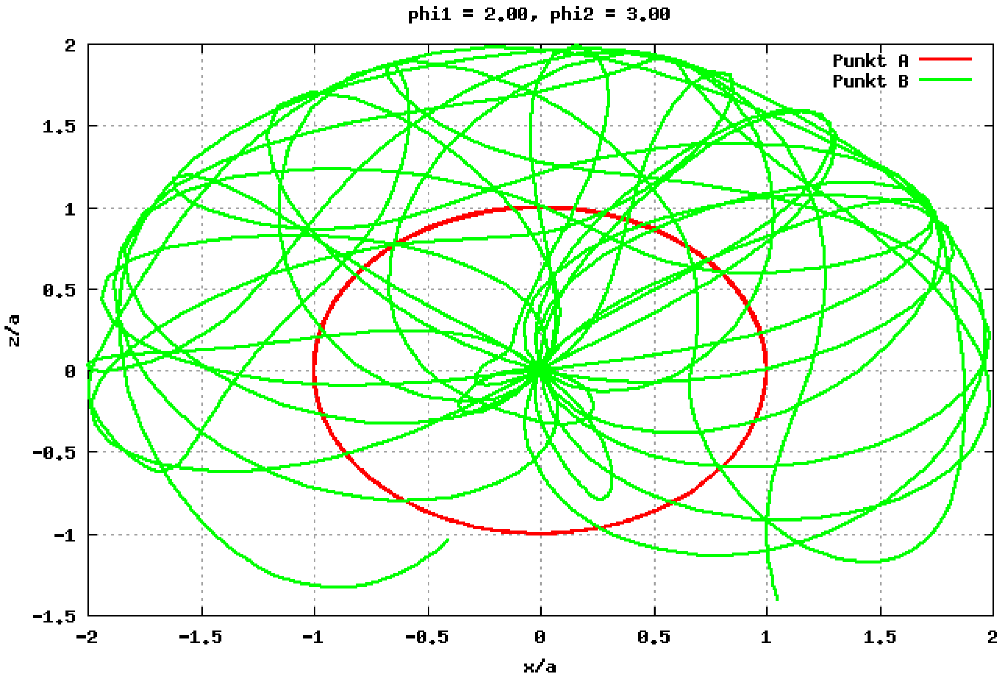
Fall a):


phi1 = 1.00, phi2 = 2.00
phi1
phi2
3
2
1
0
-1
-2
-3
phi
0
2
4
6
8
10
t [s]


phi1 = 1.00, phi2 = 2.00
Punkt A
Punkt B
2
1.5
1
0.5
0
z/a
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x/a

Fall b):





Das Doppelpendel zeigt im Fall b) chaotisches Verhalten.